Exercise Sheet: Lecture 2 Coding Theory and GUAVA

Michel Lavrauw

GAP Days Spring 2025, VUB

Exercises

- 1. Constructing a linear code from its generator matrix.
 - (a) Define a binary linear code using the generator matrix:

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

- (b) Compute its length, dimension, and minimum distance.
- (c) Encode the message (1, 0, 1).
- (d) Decode the received word (1, 1, 1, 1, 0).
- 2. Code from the Fano plane.
 - (a) Build the *incidence matrix* of points and planes in PG(2, 2). This is a 7×7 -matrix whose columns are indexed by the points of PG(2, 2), and whose rows are indexed by the lines of PG(2, 2). The entry in position (i, j) is 1 if the *i*-th point is on the *j*-th line, and 0 otherwise.
 - (b) Use the rows of the incidence matrix as generators to define the code over GF(2).
 - (c) Determine the parameters of this code.
- 3. Reed-Solomon Codes
 - (a) Construct a [7,3] Reed–Solomon code C over \mathbb{F}_8 .
 - (b) Find its minimum distance.
 - (c) Encode a message of your choice.
 - (d) Introduce one error in the codeword and attempt to decode it.
 - (e) Define the set \mathcal{A} of points defined by the column vectors of a generator matrix of \mathcal{C} .

- (f) Write a function which takes as input an arc \mathcal{A} of PG(2,q), and returns a code with generator matrix the matrix whose columns are coordinate vectors of the points of \mathcal{A} .
- 4. Explore cyclic codes
 - (a) Try constructing a cyclic code over \mathbb{F}_3 of length 9.
 - (b) Can you find a generator polynomial for it?
 - (c) Find a generator polynomial for the dual code.
 - (d) Analyse the code's parameters and error-correcting capabilities.
- 5. Codes from projective spaces.
 - (a) Build the incidence matrix of points and planes in PG(3,5). (Use the FinInG package.)
 - (b) Use the rows of the incidence matrix as generators to define the code over GF(5).
 - (c) Determine the parameters of this code.
- 6. BCH codes.
 - (a) Construct a BCH code
 - (b) Use the rows of the incidence matrix as generators to define the code over GF(5).
 - (c) Determine the parameters of this code.