Intersecting Codes in the Rank Metric

Martin Scotti

Université Paris 8 - LAGA

Seventh Irsee Conference
02/09/2025

Joint work with Daniele Bartoli, Martino Borello, and Giuseppe Marino

The rank metric

Let $x = (x_1, \ldots, x_n) \in \mathbb{F}_{q^m}^n$ and Γ an $\mathbb{F}_q$-basis of $\mathbb{F}_{q^m}$.

$$\mathrm{Mat}_\Gamma(x) = \begin{pmatrix} x_{1,1} & \cdots & x_{n,1} \\ x_{1,2} & \cdots & x_{n,2} \\ \vdots & \vdots & \vdots \\ x_{1,m} & \cdots & x_{n,m} \end{pmatrix}.$$

Definition.

An $[n,k,d]_{q^m/q}$-*code* is an $\mathbb{F}_{q^m}$-linear subspace $\mathcal{C} \subset \mathbb{F}_{q^m}^n$ of dimension $\dim_{\mathbb{F}_{q^m}}(\mathcal{C}) = k$ and minimum distance

$$d(\mathcal{C}) = \min_{c \in \mathcal{C} \setminus \{0\}} \mathrm{rk}(c).$$

The support is

$$\sigma_\Gamma(x) = \mathrm{rowspan}(\mathrm{Mat}_\Gamma(x)) \subseteq \mathbb{F}_q^n.$$

The support does not depend on the choice of Γ.

Singleton Bound.

Let $\mathcal{C}$ be a $[n,k,d]_{q^m/q}$-code in the rank metric. Then

$$km \leqslant \max(m,n) \cdot (\min(m,n) - d + 1).$$

Codes reaching the Singleton bound are called MRD codes.

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Definition.

A linear code in the rank metric $\mathcal{C}$ is **intersecting** if $\forall c, c' \in \mathcal{C}\backslash\{0\}$

$$\sigma(c) \cap \sigma(c') \neq \{0\}.$$

In the Hamming metric, a code $\mathcal{C}$ is intersecting if

$$\forall c, c' \in \mathcal{C}\backslash\{0\}, \quad \exists 1 \leqslant i \leqslant n, \quad c_i \neq 0 \wedge c_i' \neq 0.$$

Examples.

An $[n, k, d]_q$-code with $2d > n$ is Hamming-metric intersecting. An $[n, k, d]_{q^m/q}$-code with $2d > n$ is rank-metric intersecting.

Proposition. (Bartoli, Borello, Marino, S. - 202+)

An **MRD code** over $\mathbb{F}_{q^m}$ of length $n \leqslant m$ is rank-metric intersecting iff $2d > n$.

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Motivations (We are interested in the q-analogue of these).

- **(2, 1)-separating systems** (Friedman, Graham, Ullman, 1969).
- Problems of critical race-free coding for states of **discrete automata**.
- **Communication** over an *AND* channel (Cohen, Lempel, 1985).
- Application to **hash functions** (Körner, Simonyi, 1988).
- When $q = 2$ an intersecting code is a **minimal code** (related to Massey's **secret sharing schemes**, 1993).
- Frameproof codes for **digital fingerprint** (Boneh, Shaw, 1995).
- **Oblivious transfer protocols** (Brassard, Crepeau, Santha, 1996).
- **Genetics problems** (Sagalovich, Chilingarjan, 2009).
- Information on the 2-wise **Davenport constants** (Plagne, Schmid, 2011).
- Information on the generalized 2-wise Davenport constants (Borello, Schmid, S., 2024) and hence on **factorization in Dedekind domains**.

Theorem (Bartoli, Borello, Marino, S. - 202+).

$$\mathcal{C} \subseteq \mathbb{F}_{q^m}^n \text{ is rank-metric intersecting}$$

$$\Updownarrow$$

$\forall A \in \mathrm{GL}(n, q)$, $\mathcal{C}A := \{cA : c \in \mathcal{C}\}$ is Hamming-metric intersecting (i.e. two nonzero codewords c, c' always share a nonzero coordinate $c_i \neq 0, c'_i \neq 0$)

The parameters of a Hamming-metric intersecting $[n, k, d]_q$-code verify $n \geqslant 2k - 1$.

Corollary.

$$\mathcal{C} \subseteq \mathbb{F}_{q^m}^n \text{ is rank-metric intersecting of dimension } k$$

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The Geometry of Linear Rank-Metric Codes

- $G \in \mathcal{M}_{k\times n}(\mathbb{F}_{q^m})$ s.t. $\mathcal{C} = \mathrm{rowsp}(G)$.
- $\mathcal{C}$ **nondegenerate**: $\mathbb{F}_q$-span of the columns of G has $\mathbb{F}_q$-dimension n.

$$\mathcal{U} = \langle \text{col.s of } G\rangle_{\mathbb{F}_q} \qquad \begin{bmatrix} g_{1,1} & g_{1,2} & \cdots & g_{1,n} \\ \vdots & \vdots & & \vdots \\ g_{k,1} & g_{k,2} & \cdots & g_{k,n} \end{bmatrix}$$

$$\mathrm{wt}_{\mathrm{rk}}(\boldsymbol{x}G) = \boldsymbol{n} - \mathbf{dim}_{\mathbb{F}_q}(\mathcal{U} \cap \langle \boldsymbol{x}\rangle^{\perp}_{\mathbb{F}_{q^m}}).$$

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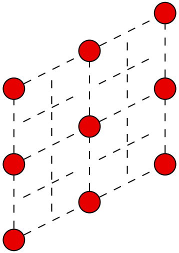

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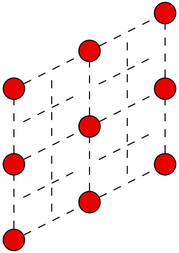

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q-SYSTEMS

Definition.

An $[n,k,d]_{q^m/q}$-**system** is an $\mathbb{F}_q$-linear vector subspace $\mathcal{U} \subset \mathbb{F}_{q^m}^k$ such that $\langle \mathcal{U} \rangle_{\mathbb{F}_{q^m}} = \mathbb{F}_{q^m}^k$, such that $\dim_{\mathbb{F}_q}(\mathcal{U}) = n$ and for any $\mathbb{F}_{q^m}$-linear hyperplane $\mathcal{H} \subset \mathbb{F}_{q^m}^k$

$$\dim_{\mathbb{F}_q} \mathcal{U} \cap \mathcal{H} \leqslant n - d.$$

q-systems $\leftrightarrow$ nondegenerate rank-metric codes

Definition.

The **weight** of a subspace $\mathcal{S} \subset \mathbb{F}_{q^m}^k$ is

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The geometric interpretation of intersecting codes

Definition.

A q-system U is *t-spannable* if there exist t $\mathbb{F}_{q^m}$-linear hyperplanes $\mathcal{H}_1, \ldots, \mathcal{H}_t$ of $\mathbb{F}_{q^m}^k$ of such that

$$U = \langle U \cap \mathcal{H}_1 \rangle_q \oplus \ldots \oplus \langle U \cap \mathcal{H}_t \rangle_q .$$

- $\mathcal{C} \subseteq \mathbb{F}_{q^m}^n$ nondegenerate of dimension k.
- $G \in \mathbb{F}_{q^m}^{k \times n}$ s.t. $\mathcal{C} = \mathrm{rowsp}(G)$ a generator matrix, and $\mathcal{U}$ the corresponding q-system.

Theorem (Bartoli, Borello, Marino, S. - 202+).

$$\mathcal{C} \text{ is rank-metric intersecting} \\ \Updownarrow \\ \mathcal{U} \text{ is not 2-spannable.}$$

This does not depend on the choice of generator matrix !

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If $\mathcal{U}$ is a q-system that is not 2-spannable then for every hyperplane

$$n - m \leqslant \mathrm{wt}_{\mathcal{U}}(\mathcal{H}) \leqslant n - k.$$

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If $\mathcal{C}$ is rank-metric intersecting then

$$2k - 1 \leqslant n \leqslant 2m - 3 \text{ and } k \leqslant d \leqslant m.$$

There exists an intersecting code if

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What if $2m - 2k + 2 \leqslant n \leqslant 2m - 3$?

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THE SPECIAL CASE $k = 3,\ m = 5$

The possible values for n are $n = 5$ (MRD codes), $n = 6$, and $n = 7$.

THEOREM (BARTOLI, BORELLO, MARINO, S. - 202+).

There exists no rank-metric intersecting code with parameters $[7, 3]_{q^5/q}$.

In the special case $q = 2$:

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- We defined **intersecting codes** in the rank metric and their geometric counterparts, i.e. **non-2-spannable** q-systems.
- We presented some **bounds** on the rank of **non-2-spannable** q-systems.

Open questions.

- Further investigations of the "grey zone" $2m - 2k + 2 \leqslant k \leqslant 2m - 3$ (current work when k=3)
- What happens in the sum-rank metric ?
- Applications ?

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